Subject. (L.04) Date.

Integration by parts

$$\int u\,dv + \int v\,du = \int d[uv]$$

$$\boxed{\int u\,dv = uv - \int v\,du} \Longrightarrow \text{بدون حفظ}$$

$$\int \frac{\ln X}{u}\, d\frac{X}{v} = \ln X \cdot X - \int X\, d(\ln X)$$

$$\begin{cases} \frac{d}{dX}\ln X = \frac{1}{X} \\ d\ln X = \frac{1}{X}\,dX \end{cases}$$

$$= X\ln X - \int \cancel{X}\,\frac{1}{\cancel{X}}\,dX$$

$$= X\ln X - X + C \quad \#$$

$$\int X \cos X\, dX$$

$$\int X\, d\sin X = X\sin X - \int \sin X\, dX$$

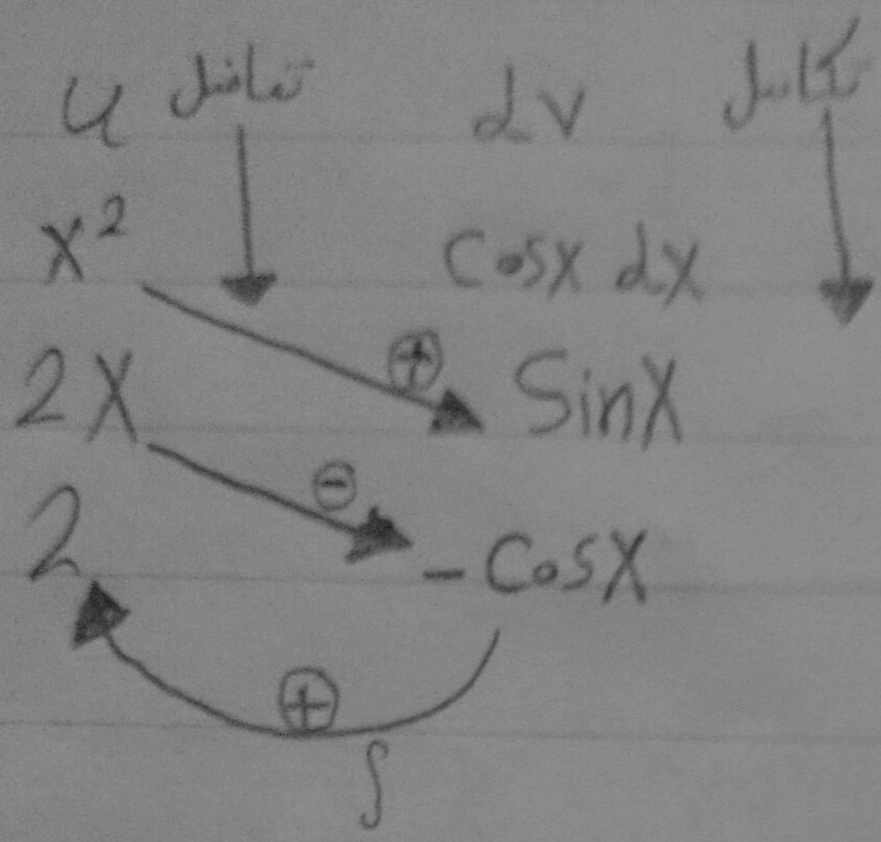
$$= X\sin X - (-\cos X) + C$$

$$= X\sin X + \cos X + C$$

Subject

Date

$$I = \int X^2 \cos X \, dX$$



$$I = X^2 \sin X - 2X(-\cos X) + \int 2(-\cos X) \, dX$$

$$= X^2 \sin X + 2X \cos X - 2 \int \cos X \, dX$$

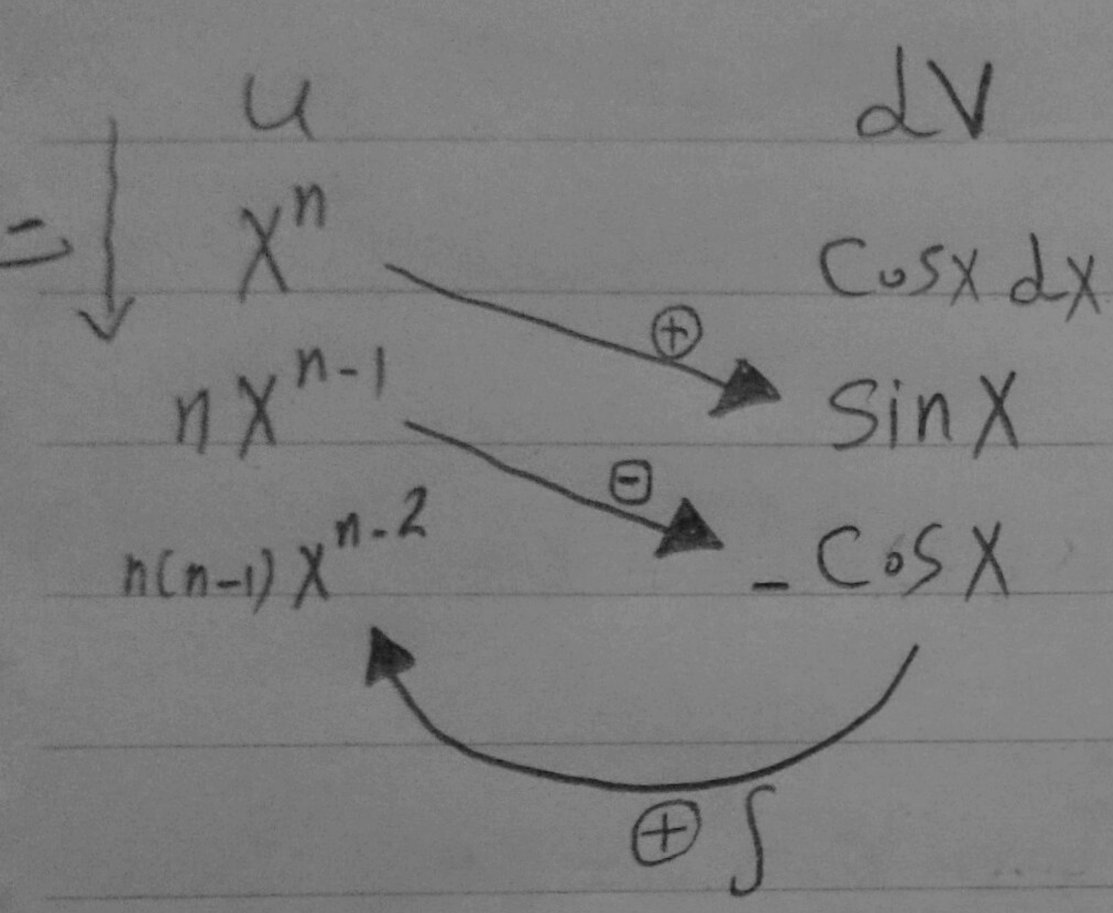
$$= X^2 \sin X + 2X \cos X - 2 \sin X + C \quad \#$$

Subject : Date :

$$\text{Ex: } I_n = \int x^n \cos x \, dx \qquad , n = 0, 1, 2, \dots$$

Sol ↳

$$I_n = \int x^n \cos x \, dx$$



$$I_n = x^n \sin x - n x^{n-1}(-\cos x) + \int n(n-1) x^{n-2}(-\cos x) \, dx$$

$$= x^n \sin x + n x^{n-1} \cos x - n(n-1) I_{n-2} \quad \#\#$$

$$I_{n=2} = x^2 \sin x + 2x \cos x - 2(1) I_0$$

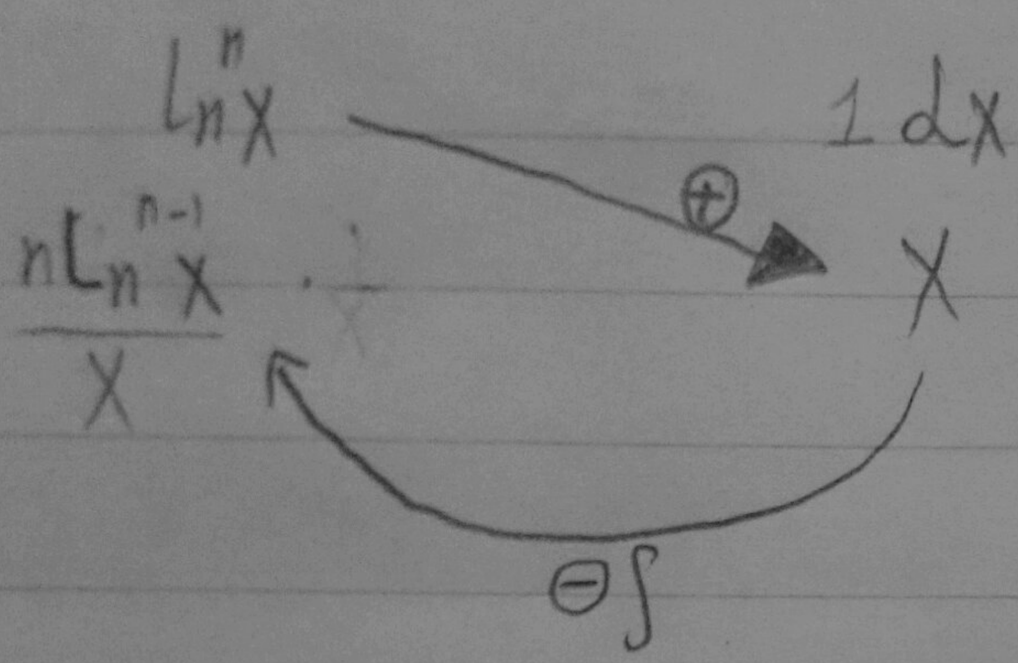
$$= x^2 \sin x + 2x \cos x - 2 \int x^0 \cos x \, dx$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C \quad \#\#$$

To you !! $\int x^3 \sin x \, dx$, $\int x^n \sin x \, dx$

Pg [39] EX.: $\int [\ln x]^n \, dx$

$$I_n = \int \ln^n x \, dx$$



$$I_n = x \ln^n x - \int \frac{n \ln^{n-1} x}{x} \cdot x \, dx$$

$$= x \ln^n x - n \int \ln^{n-1} x \, dx$$

$$= x \ln^n x - n I_{n-1} \quad **$$

$\int \ln^1 x \, dx = -- ??$

Subject. Date.

Pg (37) EX: $\int e^x \sin x \, dx$

u	dv
$\sin x$ ↓ تفاضل	$e^x dx$ ↓ تكامل
$\cos x$	e^x ⊕
$-\sin x$	e^x ⊖
	$\int$ ⊕

$$I = e^x \sin x - e^x \cos x + \int (-\sin x) e^x \, dx$$

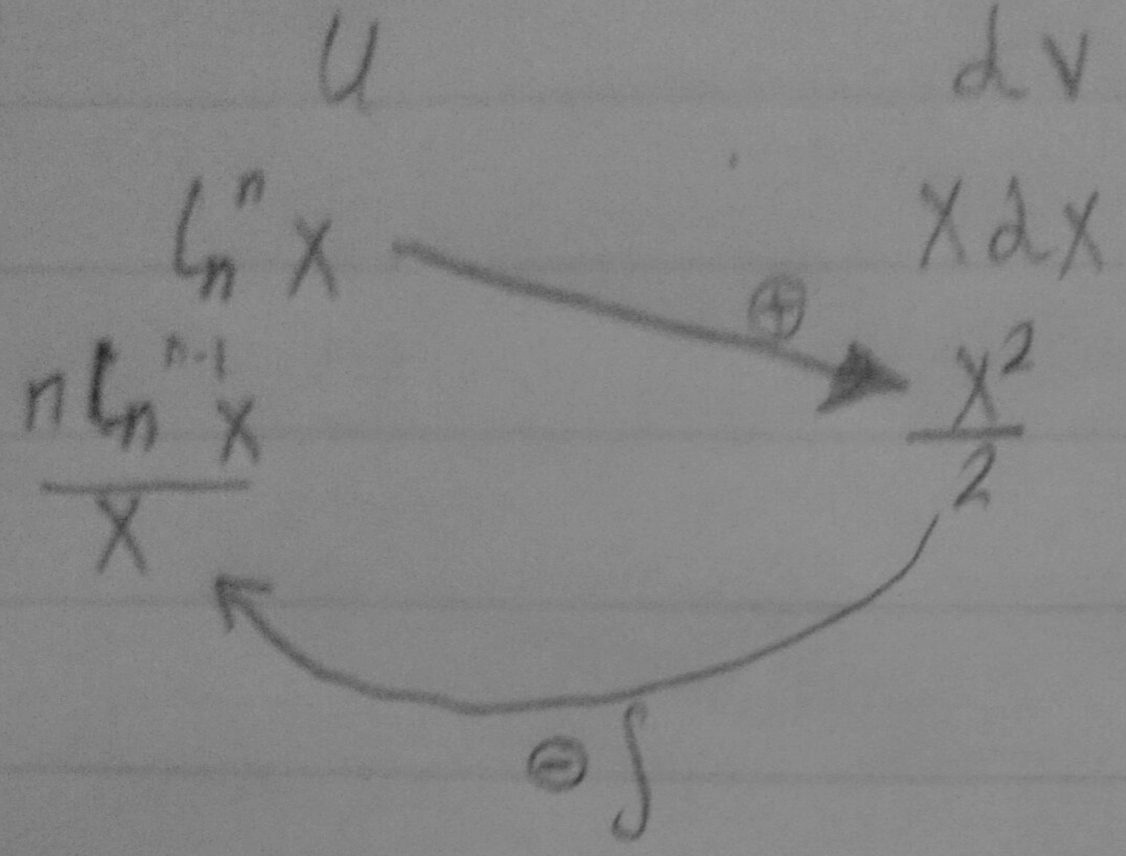
$$I = e^x \sin x - e^x \cos x - I$$

$$2I = e^x \sin x - e^x \cos x$$

$$I = \frac{e^x}{2} (\sin x - \cos x) + C$$

To you $\int e^x \cos x$

Ex: $\int x \ln^n x \, dx$



$$I_n = \frac{x^2}{2}\ln^n x - \int \frac{nx}{2}\ln^{n-1} x \, dx$$

$$= \frac{x^2}{2}\ln^n x - \frac{n}{2}\int x \ln^{n-1} x \, dx$$

$$= \frac{x^2}{2}\ln^n x - \frac{n}{2} I_{n-1} \quad \#\#$$

$$I_1 = \int x \ln x \, dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2} I_0 = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$$I_0 = \int x \, dx = \frac{x^2}{2}$$

Subject. Date.

Remember that

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

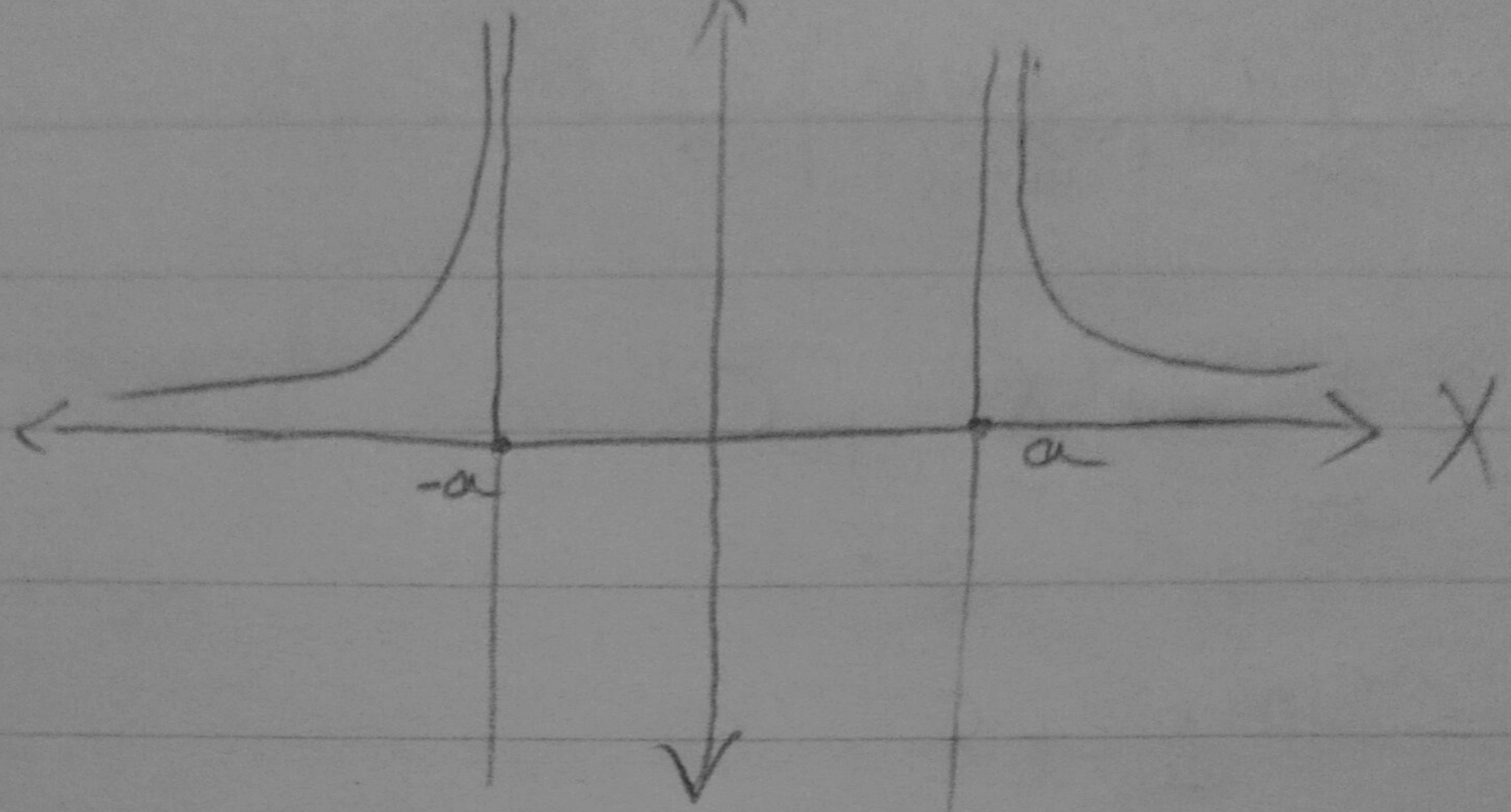
$$\int \frac{dx}{\sqrt{a^2+x^2}} = \begin{cases} \rightarrow \sinh^{-1} \frac{x}{a} + C \\ \text{OR} \\ \rightarrow \ln \frac{x+\sqrt{a^2+x^2}}{a} + C \end{cases}$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \begin{cases} \rightarrow \cosh^{-1} \frac{x}{a} + C \quad , x > a \\ \text{OR} \\ \rightarrow \ln \frac{|x+\sqrt{x^2-a^2}|}{a} + C \quad , x \neq \pm a \end{cases}$$

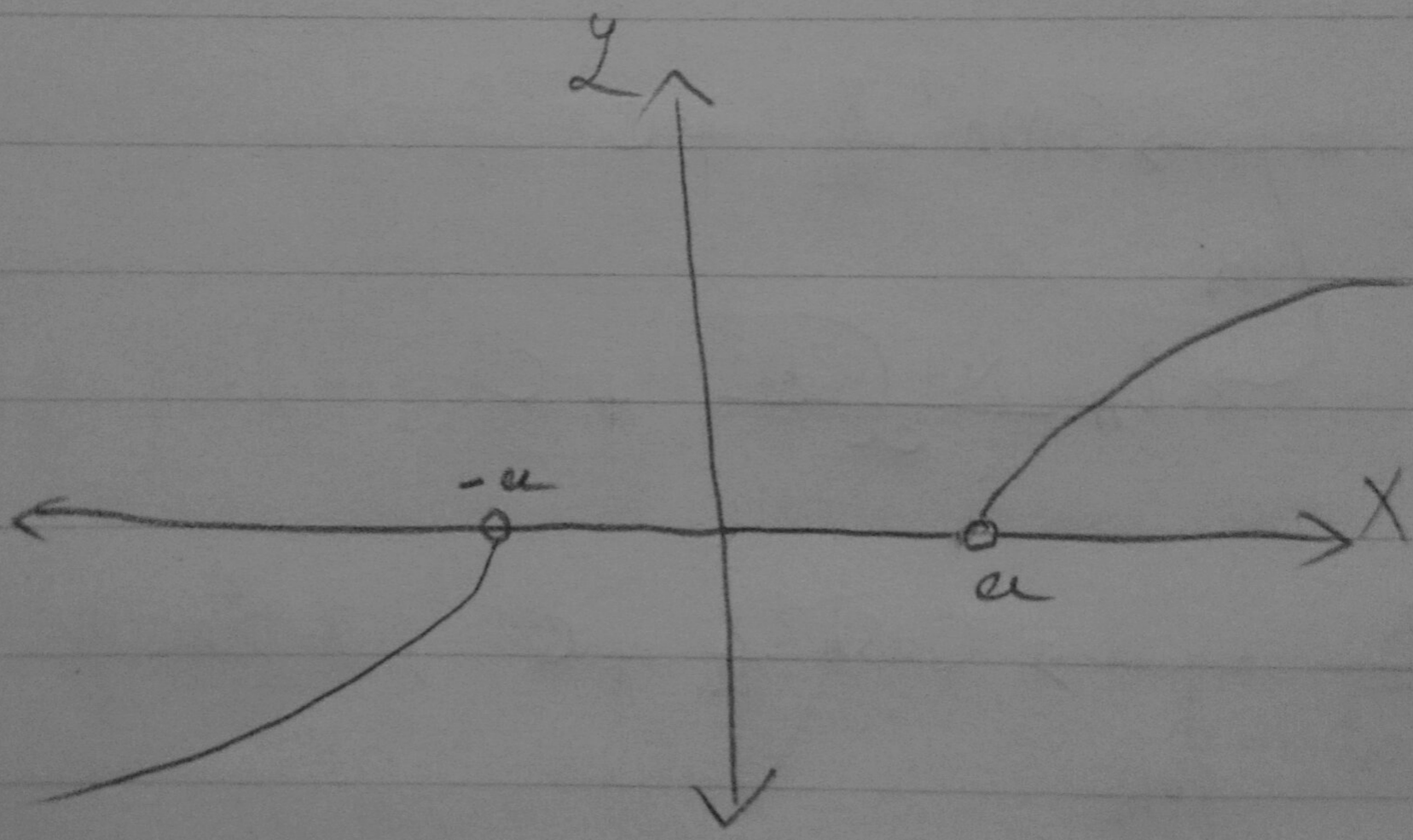
Subject. Date.

$$(\cosh^{-1}\frac{x}{a})' = \frac{1}{\sqrt{x^2-a^2}}, \quad x > a$$

$\frac{1}{\sqrt{x^2-a^2}}$



$\cosh^{-1} x$



Subject.

Date.

Ex: $\int_0^1 \sin^{-1} x \, dx$

Pg (38)

Sol

$$\int \frac{u'}{2\sqrt{u}} dx = \sqrt{u}$$

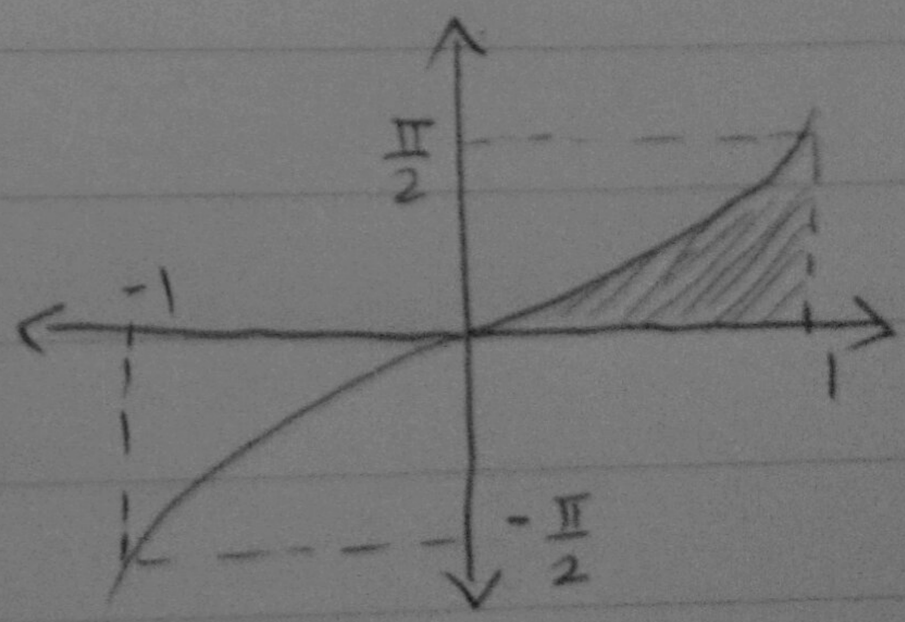
u	dv
$\sin^{-1} x$	dx
$\frac{1}{\sqrt{1-x^2}}$	x

⊕ ⊖ ∫

$$I = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$= x \sin^{-1} x + \int \frac{(-2)x}{(2)\sqrt{1-x^2}} dx$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + C$$

$$\int_0^1 = \left(1 \sin^{-1}(1) + 0\right) - \left(1\right)$$

$$= \frac{\pi}{2} - 1 = .57$$ وهو المطلوب